



PRIMARY SEVEN MATHEMATICS SELF STUDY ACTIVITIES- SET THREE

TOPIC : NUMBER PATTERNS AND SEQUENCE

SUB TOPIC : Finding consecutive numbers

Consecutive numbers

These are numbers which follow each other in order.

These include;

- Whole numbers e.g. $\{0, 1, 2, 3, 4, 5, 6, 7\dots\}$
- Counting numbers e.g. $\{1, 2, 3, 4, 5, 6, 7\dots\}$
- Natural numbers e.g. $\{1, 2, 3, 4, 5, 6\dots\}$
- Integers e.g. $\{-2, -1, 0, 1, 2, 3\dots\}$
- Even numbers e.g. $\{0, 2, 4, 6, 8\dots\}$
- Odd numbers e.g. $\{1, 3\dots\}$

Finding consecutive counting numbers/ natural numbers/ integers/ whole numbers

- The pattern of whole numbers, counting numbers, natural numbers and integers is 1(one) i.e. to get the next number, you keep on adding 1 e.g. If the 1st counting number is 2, then to get the next counting number it will be $2 + 1$ giving us 3 and if we want the next counting number, we add 1 to 3 to get 4. This makes 2, 3, 4 in that order.
- In a situation when they have not specified a counting number to start with, we assign it (the first coming number) any letter of our choice, for instance if the 1st counting number is k , then the second will be $k+1$, the next will be $k + 1 + 1$ which is the same as $k + 2$.

Example 1

The sum of four consecutive counting numbers is 18

- a) Find the numbers
- b) Find their range

Let the 1st counting number be x.

1 st no.	2 nd no.	3 rd no.	4 th no.	Sum
x	x+1	x+2	x+3	18

$$x + x + 1 + x + 2 + x + 3 = 18$$

$$x + x + x + x + 1 + 2 + 3 = 18$$

$$4x + 6 = 18$$

$$4x + 6 - 6 = 18 - 6$$

$$\frac{4x}{4} = \frac{12}{4}$$

$$x = 3$$

Then substitute

1 st no.	2 nd no.	3 rd no.	4 th no.
x	x+1	x+2	x+3
3	3 + 1	3 + 2	3 + 3
	4	5	6

Therefore; The numbers are 3, 4, 5 and 6

Since we don't know the numbers we assign the first number any letter, in this case we assign letter x

Represent the consecutive numbers in terms of x

Form an equation for the sum in terms of the unknown letter and equate to the given sum, so that you can find the value of the unknown letter

b) Range = highest – lowest

$$= 6 - 3$$

$$= 3$$

Examples 2

1. The sum of three consecutive counting numbers is 36. Find the numbers?

Let the 1st counting number be k

1 st no.	2 nd no.	3 rd no	Sum
k	$k+1$	$k+2$	36

$$k + k+1 + k+2 = 36$$

$$k + k + 1 + k + 2 = 36$$

$$k + k + k + 1 + 2 = 36$$

$$3k + 3 = 36$$

$$3k + 3 - 3 = 36 - 3$$

$$\frac{3k}{3} = \frac{33}{3}$$

$$k = 11$$

Then substitute to get the numbers

1 st no.	2 nd no.	3 rd no
k	$k+1$	$k+2$
11	11+1	11+2
	12	13

Therefore, the numbers are 11, 12 and 13.

Work to do

1. The sum of 3 consecutive counting numbers is 21. What are the numbers?
2. Find 3 consecutive counting numbers whose sum is 51
3. The sum of 3 consecutive counting numbers is 72. Express the largest number in standard form.
4. The sum of three consecutive counting numbers is 93. Find their range.
5. Akoth planted 300 coffee seedlings in her garden in three days. If she planted 10 more seedlings on the next day,
 - a) How many seedlings did she plant on the first day?
 - b) Find the total number of seedlings she planted in the last two days.

Form an equation of the sum in terms of the unknown letter. This helps us get the value of the unknown letter used.

LESSON 2

Finding consecutive odd numbers and even numbers

Note: Consecutive numbers are numbers which follow each other in order, for example

Consecutive odd numbers: 1, 3, 5, 7, 9, 11, they either increase by 2 or decrease by 2.

Consecutive even numbers: 0,2,4,6,8,10,, they either increase by 2 or decrease by 2.

- Therefore the difference between consecutive odd numbers and even numbers is 2. i.e you get the next number by adding 2. If the first odd number is 1, to get the next odd numbers you keep on adding 2 to the result e.g. $1 + 2 = 3$, $3 + 2 = 5$, $5 + 2 = 7$ and so on. For even numbers if the 1st one is 0 and you add 2, you get 2, keep on adding 2 to the result you get even numbers.
- In case the odd or even numbers are not specified we can use letters to represent them. If the first one is k , the next will be $k + 2$, the next will be $k + 2 + 2$ which is $k + 4$ etc.

Example

The sum of three consecutive odd numbers is 51.

- Find the numbers
- Workout the product of the 1st and 3rd odd numbers

Let the first odd number by y

1 st no.	2 nd no.	3 rd no	Sum
y	$y+2$	$y+4$	51

$$y + y + 2 + y + 4 = 51$$

$$y + y + y + 2 + 4 = 51$$

$$3y + 6 = 51$$

$$3y + 6 - 6 = 51 - 6$$

$$\frac{3y}{3} = \frac{45}{3}$$

$$**y = 15**$$

Form an equation in terms of y and find the value of the unknown

Substitute for the value of y to get the wanted odd numbers

1 st no.	2 nd no.	3 rd no
y	y+2	y+4
15	15 + 2	15 + 4
	17	19

Therefore; the numbers are 15, 17 and 19

b) Product of the 1st and 3rd no

$$\begin{array}{r}
 419 \\
 \times 15 \\
 \hline
 195 \\
 +190 \\
 \hline
 285
 \end{array}$$

Example 2

The sum of 5 consecutive even numbers is 140. If the first is P.

- i) Find the numbers
- ii) Calculate their range

The 1st no. is P.

1 st no.	2 nd no.	3 rd no.	4 th no.	5 th no.	Sum
P	P+2	P+4	P+6	P+8	140

$$P + P + 2 + P + 4 + P + 6 + P + 8 = 140$$

$$P + P + 8 + 8 + 2 + 4 + 6 + 8 = 140$$

$$5P + 20 = 140$$

$$5P + 20 - 20 = 140 - 20$$

$$\frac{5P}{5} = \frac{120}{5}$$

$$P = 24$$

1 st no.	2 nd no.	3 rd no.	4 th no.	5 th no.
P	P+2	P+4	P+6	P+8
24	24 + 2	24 + 4	24 + 6	24 + 8
24	26	28	30	32

Then substitute for p to get the wanted even numbers

b) Range

$$\begin{aligned} \text{Range} &= \text{Highest} - \text{Lowest} \\ &= 32 - 24 \\ &= 8 \end{aligned}$$

Examples 3

The sum of three consecutive even numbers is 24. If the middle number is p, find the numbers.

1 st no.	2 nd no.	3 rd no.
p-2	p	p+2

Note: p in this case represents the middle number

$$p-2+p+p+2 = 24$$

$$p+p+p-2+2 = 24$$

$$3p = 24$$

$$\frac{3p}{3} = \frac{24}{3}$$

$$p = 8$$

1 st	2 nd	3 rd
p-2	p	p+2
8-2	8	8+2
6	8	10

Therefore, the numbers are 6, 8, 10

Activity

- The sum of three consecutive even numbers is 42. Find the numbers.
- The sum of three consecutive odd numbers is 45. If the largest is y, find the numbers.
- The sum of three consecutive even numbers is 48. Taking the median as n, find their range.
- The median of three consecutive numbers is 14.
 - Find the numbers.
 - Find their sum
- Kayera planted 480 eucalyptus seedlings in 3 days. If she planted 20 more seedlings on the next day.
 - How many seedlings did she plant on the first day?
 - Express the number of seedlings planted on the second and third day in scientific notation.

LESSON 3

Representing prime factors on a Venn diagram

Competences:

The Learner should be able to:

- Prime factorise the given numbers and represent them on a Venn diagram.
- Divide the given numbers and write their prime factors in set notation

Example 1

Represent the prime factors of 36 and 30 on a Venn diagram

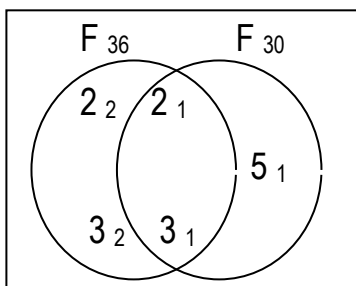
2	36
2	18
3	9
3	3
	1

2	30
3	15
5	5
	1

$$F_{30} = \{2_1, 3_1, 5_1\}$$

$$F_{36} = \{2_1, 2_2, 3_1, 3_2\}$$

$$C.F. = \{2_1, 3_1\}$$



GCF

$$2 \times 3 = 6$$

LCM

$$2 \times 2 \times 3 \times 3 \times 5$$

$$12 \times 15 = 180$$

	1	2
x	1	5
	6	0
	<u>+1</u>	<u>2</u>
		180

- Put the Common factors in the intersection
- Then write the remaining prime factors in the respective regions
- When tasked to get GCF (Greatest Common Factor) multiply all the factors in the intersection
- When tasked to get LCM (Lowest Common Multiple) multiply all the factors in the union

Work to do

a) Prime factorize the following numbers in set notation and represent the prime factors on a Venn diagram.

1. a) 24 and 30
b) Use the Venn diagram to find their GCF and LCM
2. a) 30 and 48
b) Use the Venn diagram to find their GCF and LCM
3. a) 48 and 60
b) Use the Venn diagram to find their GCF and LCM
4. a) 36 and 54
b) Use the Venn diagram to find their GCF and LCM

Express in set notation form and show on a Venn diagram

5. 2×3 and $3^2 \times 5$
6. $3^2 \times 5$ and $2^5 \times 3$
7. $2^1 \times 3^1$ and $3^2 \times 5$
8. $2 \times 3^2 \times 7$ and $2^2 \times 3 \times 5$

LESSON 4

Competences: more about GCF and LCM

The learner should be able to:

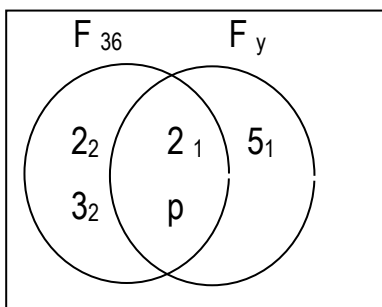
- Find the GCF and LCM using a factor set.
- Find the unknown prime factors on a Venn diagram.

Interpreting information on a factor set

1. To find the GCF or HCF, we consider the product of the intersection set.
2. To find the LCM, we consider the product of the union set.
3. The position of the unknown determines the way its value will be found.
4. Unknowns inside the Venn diagram are found by division and those outside are found by multiplication.

Examples.

1. Study the Venn diagram below and use it to answer questions that follow



a) Find the value of p

$$\{2_1, p, 3_2, 2_2\} = F_{36}$$

$$2 \times p \times 3 \times 2 = 36$$

$$12p = 36$$

$$\frac{12p}{12} = \frac{36}{12}$$

$$p = 3$$

c) Find the HCF of 36 and y .

H.C.F = Product of the intersection

$$= 2 \times 3$$

$$= 6$$

b) Find the value of y .

$$F_y = \{5_1, 2_1, 3_1\}$$

$$= 5 \times 2 \times 3$$

$$= 30$$

d) work out the L.C.M of 36 and y

L.C.M = Product of the union

$$= 3 \times 2 \times 3 \times 2 \times 5$$

$$= 180$$

2. Given that $F_p = \{2_1, y, 5_2\}$ and $F_q = \{2_1, y, 5_2\}$. If the Highest common factor (HCF) of p and q is 10, find the value of y.

$$\{2_1, y\} = 10$$

$$2 \times y = 10$$

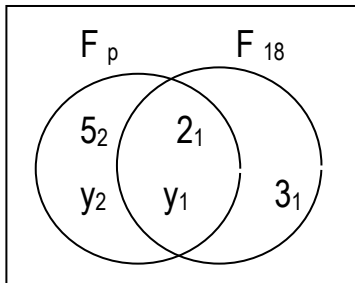
$$2y = 10$$

$$\frac{2y}{2} = \frac{10}{2}$$

$$y = 5$$

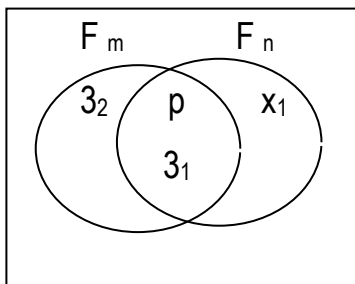
Work to do

1. Study the Venn diagram below and use it to answer questions that follow.



- Find the value of y.
- Find the value of p.
- Work out the greatest common divisor (GCD) of p and 18.
- Calculate the lowest common multiple (LCM) of p and 18.

2. Study the Venn diagram below and use it to answer questions that follow.



- Given that the GCF of n and m is 6, find the value of p.
- Find the value of x if the LCM of n and m is 90.
- Calculate the value of m and n.

3. Given that $p = \{2_1, 2_2, 2_3, 3_1\}$ and $q = \{2_1, 3_1, 5_1\}$.
- Find the value of p .
 - Find the value of q .
 - Calculate the LCM of p and q .
 - Work out the GCF of p and q .
4. Given that $18 = 2 \times 3^2$ and $24 = 2^3 \times 3$. Find the GCF using the above prime factors.

LESSON 5

Competences:

The learner should be able to:

- Apply LCM and GCF in real life situations.

Application of LCM AND GCF

Examples.

1. The LCM of two numbers is 24 and their GCF is 2. Find the second number if the first number is 8.

$$\begin{aligned} 2^{\text{nd}} \text{ number} &= \frac{LCM \times GCF}{1^{\text{st}} \text{ number}} \\ &= \frac{24 \times 2}{8} \\ &= \frac{48}{8} \\ &= 6 \end{aligned}$$

2. Find the least number of tomatoes that can be shared by 8 boys and 6 girls to leave 3 as a remainder.

2	8	6
2	4	3
2	2	3
3	1	3
	1	1

$$\begin{aligned} LCM &= (2 \times 2 \times 2) \times 3 \\ &= 8 \times 3 \\ &= 24 \end{aligned}$$

$$\begin{aligned} \text{Least no} &= LCM + \text{remainder} \\ &= 24 + 3 \\ &= 27 \end{aligned}$$

3. Find the highest number that can divide 18 and 24 without leaving a remainder.

2	18	24	GCF = 2×3 = 6
3	9	12	
	3	4	

NOTE:

The **least number** requires finding the **lowest common multiple** while the **highest number** requires to find the **greatest common factor**.

Work to do

1. Find the highest number of girls that can share 48 mangoes and 36 oranges without leaving a remainder.
2. The ratio of two numbers is 4:5. If their GCF is 8,
 - a) Find the numbers.
 - b) Calculate their LCM
3. The LCM of two numbers is 120 and their HCF is 10. If the first number is 40, find the second number.
4. In Cornerstone Junior School, two bells ring at an interval of 30min and 40mins to change lessons.
 - a) After how many minutes will they ring together again?
 - b) If they first rung together at 10.00am, at what time will they ring together again?
5. Besigye, Lukwago and Bobi wine are to report to Buganda court after every 12days, 18days and 24days respectively.
 - a) After how many days will they report to court together?
 - b) If they first reported to court together on Tuesday, on what day of the week will they report together again for the second time?
6. Find the least number of pens that can be shared by 12 boys and
7. 18 girls without leaving a remainder.
8. Airtel and MTN adverts are broadcast on NBS tv and NTV at an interval of 30min and 40mins respectively during the course of the day.
 - a) After how many hours will they be broadcast together for the very first time?
 - b) How many times was the airtel advert broadcast by the time both adverts appeared on both stations for the first time together?

LESSON 6

Number systems; are the different types of numbers we use.

Competences:

The learner should be able to:

- Define the different types of numbers
- Identify the properties of numbers
- Identify the relationships of numbers

Types of numbers

a) **Square numbers**

- Square numbers are numbers obtained by multiplying counting numbers by self two times.

Examples

$$1 \times 1 = 1$$

$$2 \times 2 = 4$$

$$3 \times 3 = 9$$

$$4 \times 4 = 16$$

$$5 \times 5 = 25$$

$$6 \times 6 = 36$$

$$7 \times 7 = 49$$

$$8 \times 8 = 64$$

$$9 \times 9 = 81$$

$$10 \times 10 = 100$$

Etc.

- Square numbers can also be obtained by adding consecutive odd numbers.

Odd numbers	Square numbers
1	1
1 + 3	4
1 + 3 + 5	9
1 + 3 + 5 + 7	16
1 + 3 + 5 + 7 + 9	25
1 + 3 + 5 + 7 + 9 + 11	36
1 + 3 + 5 + 7 + 9 + 11 + 13	49

- Square numbers include **1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121... Etc.**

Cube numbers

Cube numbers are numbers obtained by multiplying counting numbers by self three times.

Counting numbers	Cube numbers
1 x 1 x 1	1
2 x 2 x 2	8
3 x 3 x 3	27
4 x 4 x 4	64
5 x 5 x 5	125
6 x 6 x 6	216
7 x 7 x 7	343

Work to do.

1. Given that set $Q = \{\text{Square numbers less than } 100\}$. Find $n(Q)$
2. Set $P = \{\text{the first six cube numbers}\}$. List all elements of set P .
3. Find the sum of the fourth cube number and the tenth square number.
4. Find the sum of the next two numbers in the sequence 1, 8, 27, 64, ____, ____
5. Find the next number in the sequences below.
 - a) 1, 2, 6, 15, 31, ____
 - b) 64, 32, 16, 8, ____
 - c) 81, 27, 9, 3, ____
6. Find the square of the next number in the sequence. 2, 3, 7, 16, 41, ____

LESSON 7

Composite and prime numbers

COMPETENCES

The learner should be able to;

- Define a prime number and a composite number
- List prime numbers
- List composite numbers
- Find the sum, product and difference of both prime and composite numbers.

Prime numbers

Prime numbers are numbers with only two factors.i.e. one and the number itself.

Composite numbers

Composite numbers are numbers with more than two factors.

Study and complete the table below.

Number	Factors	Type
1	{1}	Neither prime nor composite
2	{1,2}	Prime no.
3	{1,3}	Prime no.
4	{1,2,4}	Composite no.
5	_____	Prime no.
6	_____	Composite no.
7	_____	_____
8	_____	_____
9	{1,3,9}	_____
10	{1,2,5,10}	_____
11	_____	_____
12	{1,2,3,4,6,12}	Composite no
13	_____	Prime number
14	_____	_____
15	_____	_____
16	{1,2,4,8,16}	Composite no.
17	{1,17}	Prime no.

Prime numbers include; 2,3,5,7,11,13,17,19,23,29,31...etc.

Composite numbers include; 4,6,8,9,10,12,14,15,16,18,20,21,22,24,25...etc.

Note:

Both prime numbers and composite numbers have an irregular pattern. Their next number in any given sequence is found by studying the sequence and understanding the relationship or type.

Work to do.

1. List all composite numbers between 20 and 35.
2. List all the odd composite numbers between 10 and 20
3. Find the sum of the 8th prime number and the 10th composite number.
4. What is the difference between the prime numbers between 30 and 40?
5. Given that $P = \{\text{the first five prime numbers}\}$. List down all elements of set P.
6. Find the next number in the sequences below
 - a. 2, 3, 5, 7, _____
 - b. 4, 6, 8, 9, 10, _____, _____
 - c. 3, 5, 8, 13, 20, _____
 - d. Find the square of the next number in the sequence 12, 10, 9, 8, 6, _____